Classification Structures for Cognitive Maps

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Summary: The ability to create and manipulate meaningful data structures of cognitive spaces remains a problem for designers of geographic information systems. Methods to represent the inherent hierarchical structure in cognitive spaces are discussed. Several alternative scaling techniques for developing hierarchical and overlapping representations, including ordered trees, ultrametric trees, and semi-lattices, are presented and discussed. To demonstrate the differences among these three representation schemes, each of three techniques is applied to two small datasets collected on the recall of capitals or countries in Europe. The methods discussed here were chosen to illustrate the limitations of a strict, hierarchical representation and because they have been used in the past to model cognitive spaces.

1. Introduction

The ability to create and manipulate meaningful data structures of cognitive spaces remains a problem for designers of geographic information systems. The ability to present and to interpret spatial data in a method that is consistent with the internal cognitive map of the user would lead to systems that are more flexible and will provide greater functionality in terms of cognitive spatial tasks (Hirtle and Heidorn, 1993; Mclvickij-Scott and Blades, 1992).

A common conclusion that has emerged from the research on the structure of cognitive mapping is that spatial memory is organized hierarchically, which results in processing biases and errors in judgments (Concelis, et al. 1987; Gollende, 1992; Hirtle and Jonides, 1985; McNamara, et al., 1988; Stevens and Coupe, 1978). However, as Hirtle (1995) argued recently, the claim that mental representations are inherently hierarchical is often made without providing an explicit alternative. For example, the first author and his colleagues have argued that their data is consistent with a "partially hierarchical model" (McNamara, et al., 1989) and have warned against the conclusion that only structure in a cognitive map is of a hierarchical nature (Hirtle and Jonides, 1985). While such qualifications are intriguing, they are often stated without proposing an explicit alternative. In this paper, several alternative scaling techniques for developing hierarchical and overlapping representations, including ordered trees, ultrametric trees, and semi-lattices, are considered.

2. Hierarchies

A strict hierarchy if often assumed for representing spatial concepts. For example, Stevens and Coupe (1978) showed how people consistently misjudged certain directions, such as assuming that Reno, Nevada is north and east of San Diego, California, when in fact Reno is north and west of San Diego. To account for such effects, Stevens and Coupe (1978) presented a nested, propositional model, with San Diego as part of California, Reno as part of Nevada, and California to the west of Nevada. Here, the reasoning processes occur on a hierarchical tree structure, which contains cities nested within states. Thus, a hierarchy is assumed to be formally equivalent to a rooted tree, in a graph-theoretic form. A hierarchy can be defined formally as a collection of sets such that any two sets in the collection, either one set is contained in the other or the two sets are disjoint (Alexander, 1965).

Many real-world phenomena can be represented by a tree, such as cities within states, states within countries, countries within continents, and so on. However, Hirtle (1995) argued that most attempts to force a hierarchy onto anything other than artificial examples usually fail. Gary, Indiana, in terms of influences, transportation, and even time zones, is more closely associated with Chicago than with the rest of Indiana. Lake Tahoe represents a single geographical "neighborhood" that lies in both California and Nevada. Such examples might be considered noise in the data to be ignored. However, in discussing the structure of cities, Alexander (1965) has argued that a natural city is by nature not hierarchical, but contains overlapping clusters that are better represented in semi-lattice. Hirtle (1995) explored this hypothesis by examining a small subsample of a larger dataset. Here, we expand on this analysis by including the entire dataset and examining alternative distance metrics. We begin by constraining two partially hierarchical structures, ordered trees and semi-lattices.

3. Ordered Trees

A technique that has proven useful for uncovering hierarchical structure in cognitive maps has been that of the ordered tree algorithm for free-recall data (Hirtle and Jonides, 1985; McNamara for et al., 1989). An ordered tree is a rooted tree where the children of a node, at any level, may be ordered, as a unidirectional or bidirectional node, or unordered, as a nondirectional node. Ordered trees, as discussed here, were first introduced by Reitman and Rueter (1980) and differ from two other uses in the literature of the term. Aho, et al. (1974) define an ordered tree as one in which all children are strictly ordered from left to right. In a third use of the term, Barthelme, et al. (1986) define an ordered tree as a rooted tree where the nodes are ordered by the height of the nodes. In this paper, the discussion is restricted to the first use of the term, as defined by Reitman and Rueter (1980).

![Fig. 1: Ordered dendrogram and set inclusion diagram for ordered tree.](image)

An ordered tree is built by examining the regularities in a set of recalls over a fixed set of items. In fact, an ordered tree is a generalization that allows for some overlapping structure. As an example, the collection of sets {NH VT}, {ME NH VT}, {CN MA}, {MA RI}, {CN MA RI}, and {CN MA ME NH RI VT} can not be represented by a tree, since the sets {CN MA} and {MA RI} are overlapping and violate the definition of a hierarchy, given above. However, this collection can be represented by the ordered tree as seen in Figure 1.
One might be tempted to conclude that an ordered tree is simply a variant of non-binary tree. However, this is not the case. Note that in the previous example, a non-binary tree could be constructed by the removal of the overlapping sets \{CN MA\} and \{MA RJ\}. However by the inclusion of these two sets, along with the explicit exclusion of the set \{CN RI\}, the collection of sets can no longer be represented by a strict hierarchy. Furthermore, many cognitive and real-world relations are best seen as exactly this type of ordered structure.

4. Semi-lattices

A semi-lattice is a generalization of an ordered tree. It is defined formally as a collection of sets, such that for any two overlapping sets in the collection, the intersection of the sets is also in the collection (Alexander, 1965). Therefore, if the sets \{A B C D E F\} and \{B C E G H\} are in the collection, then the set \{B C E\} must be in the collection, as well. As an example, consider the collection of sets \{NH VT\}, \{CN MA\}, \{ME NH VT\}, \{CN MA VT\}, \{CN MA RI\}, and \{CN MA ME NH RI VT\}. Such a collection cannot be represented as either a tree or an ordered tree, but can be represented as a semi-lattice. The sets \{ME NH VT\}, \{CN MA VT\} and \{CN MA RI\} are overlapping and thus violate the definition of an ordered tree, given above. However, this collection can be represented by the graph structure shown in Figure 2.

\[
\begin{align*}
&\{CN MA ME NH RI VT\} \\
&\{ME NH VT\} \quad \{CN MA RI\} \\
&\{NH VT\} \quad \{CN MA\} \\
&\{ME\} \quad \{NH\} \quad \{VT\} \quad \{CN\} \quad \{MA\} \quad \{RI\}
\end{align*}
\]

Fig. 2: Set inclusion diagram for semi-lattice structure.

Alexander (1965) notes that planned, or what he calls "artificial," cities often are designed using a strict tree structure. Two such examples are Columbia, Maryland, where clusters of exactly five neighborhoods combine to form villages and a 1943 plan of Greater London by Abercrombie and Forshaw argues for a "large number of communities, each separated from all adjacent communities." Each community is further subdivided into neighborhoods, each with their own shops and schools. Alexander (1965) goes on to argue that a natural, living city, despite the ill-advised wishes of the urban planner, does not conform to the hierarchical structure of a tree. Rather, as post office, a local school, a social club, or water authority all serve areas of different sizes and scope. Thus, the resulting structure is better conceptualized to be that of a semi-lattice.

5. Mapping Data to Structures

5.1 Data and Trees

To demonstrate further the differences among these three representation schemes, we turn to two small datasets collected on the recall for countries in Europe. During the academic year of 1984-1985, students on two different campuses in two different European countries were asked to make an ordered list, from memory, of either all the countries in Europe, or all the capitals in Europe. No other instructions were given to the subjects. To equate the two samples, the capitals were converted into the country name for those receiving the capital task. It is further acknowledged that the capital task was harder and that exclusions might occur, not from forgetting the country, but because the subject does not know or is unsure of the name of the capital. However, these two datasets are considered only to highlight the differences between the representations discussed in this paper, and not to generalize about specific regional understanding of European geography. Furthermore, the purpose of this exercise was to explore the possible clustering that exists among countries and not the rather trivial set inclusion principle of aggregating a capital to its host country.

A group of 18 subjects in Norway, who were asked to recall countries of Europe, produced a total of 44 distinct entries. The entire set of recalls can be seen using a path-graph visualization developed by Hirtle (1991) in Figure 3. Here, the line width is proportional to the number of the times two countries were recalled simultaneously. By visually focusing on the thicker connections, several clear clusters, such as the Scandinavian countries, begin to emerge, as seen in Figure 3.

![Path graph of European countries generated by the Norwegian subjects](image)

Fig. 3: Path graph of European countries generated by the Norwegian subjects.
A group of 12 subjects in Austria, who were asked to list all the capital cities of Europe, produced a total of 32 distinct entries. The capitals were converted to the country names, and resulting complete path-graph, shown in Figure 4.

The ordered lists from each of the datasets were clustered into a strict hierarchical tree, using an average-link clustering algorithm (UPGMA). This was done using two different measures of distance, city-block and a log-based distance. The city-block metric is equivalent to stating that the distance between any two countries is proportional to the total number of intervening items between them across all the ordered lists. However, as items are further separated on the list, the actual numerical difference becomes less important. Therefore, we replicated the analysis with the logarithm of the difference. Furthermore, four countries were dropped from the analysis, due to a lack of data for calculating pairwise distances. For simplicity, only the later distance analysis is reported here. The resulting tree is shown in Figure 5.

A group of 12 subjects in Austria, who were asked to list all the capital cities of Europe, produced a total of 32 distinct entries. The ordered recalls of these 32 countries were clustered into a strict hierarchical tree, also using the average-link clustering algorithm with the city-block distance and log-based distance. The resulting tree for log-based distances is shown in Figure 6.
5.2 Ordered trees

An ordered tree might allow some overlapping relationships to emerge. Unfortunately, an immediate application of the existing ordered tree algorithm of Reitman and Fuster (1980) is not possible. The algorithm was developed to account for the strong representational structures within a single subject for a domain of interest and not to build an average structure across many subjects. Thus, the algorithm is deterministic and produces clusters that exist across all recall patterns. Within the Norwegian sample, there was not a single cluster that was common to all subjects, whereas in the Austrian sample, only the single cluster of Norway Sweden existed for all the subjects.

However, by examining subgroups of subjects within each sample, one can identify small groups of subjects with common strategies, for which one can calculate non-trivial ordered trees. Figure 7 shows one tree from a subset of the Norwegian subjects and Figure 8 shows trees from two subsets of the Austrian subjects. It is interesting to note the predominance of the home country, as expected, in each sample. In addition, the two ordered trees in Figure 8, from the Austrian sample, indicate two very different strategies, one that is geographically oriented (Figure 8a), and another that is ordered by prominence (Figure 8b). The former strategy resulted in Austria being clustered with Switzerland and Liechtenstein, whereas the latter strategy resulted in Norway being followed by France and United Kingdom.

Fig. 7: An example of an ordered tree for a subgroup of Norwegian subjects

Fig. 8: An example of ordered trees for two subgroups of Austrian subjects

Two benefits arise from the ordered tree over the strict hierarchical tree. First, any order that might exist within a cluster is preserved. This can be seen by dominance of the home countries of Norway and Austria within their respective ordered trees in Figures 7 and 8. The ordered clusters also provide examples of implicit overlapping internal clusters. For example, in Figure 7, the ordered cluster of France, Spain, Portugal is created by the two underlying, overlapping clusters of France, Spain and Portugal both of which have strong surface validity, while the excluded relationship France, Portugal has much weaker surface validity. A strict hierarchical model would imply that every pair of items in a cluster would be associated at the same level.

5.3 Semi-lattices

The final representational scheme of a semi-lattice lacks any direct method to produce, which may account for why the representation of a semi-lattice for cognitive maps has not been considered to the extent of the previous two representations. One solution would be to use the MAPCLUS algorithm to fit the ADCLUS model (Shep-
ard and Arblade, 1979) of overlapping clusters. These clusters could then provide a seed set of potential clusters to build a semi-lattice upon. An initial application of the MAPCLUS algorithm to the data from the Norwegian subjects resulted in four clusters, with only the Scandinavian cluster being distinct from the others. The data from the Austrian subjects also resulted in four overlapping clusters. One cluster consisted of northern European countries, including Scandinavia and the British Isles. The second consisted of eastern European countries. The third cluster consisted of prominent central European countries and the final of less prominent countries. While such an analysis is promising, it is clear that any implementation of semi-lattice models will require the additional development of appropriate algorithms.

6. Conclusions and summary

In summary, a tree structure is one realization for a hierarchical structure for the representation of space. It is easily constructed and understood, but it is also a rigid structure that does not allow for overlap. Ordered trees provide an extension that allows for some degree of overlap, whereas a semi-lattice is an even richer structure that appears to be consistent with many aspects of cognitive space (Alexander, 1965). There are many other possibilities for representing spatial clusters, including additive trees (Sattath and Tversky, 1977), pseudo-hierarchies or pyramids (Diday, 1986), extended trees (Carroll and Corter, 1995), and hybrid scaling models (Carroll and Puzansky, 1980). A survey and review of the mathematical properties of many of these representations can be found in Van Cutsem (1994). The methods discussed here were chosen to illustrate the limitations of a strict, hierarchical representation and because they have been used in the past to model cognitive spaces.

As spatial information systems develop and evolve, the importance of considering alternative structures to strict hierarchical trees and the necessity of being explicit about the nature of the assumed representational structure will only increase. Spatial information systems, multimedia systems, and large information systems including the World Wide Web require a user to navigate through complex and often poorly differentiated spaces (Kim and Hirtle, 1995). The ability to generate a meaningful multi-level structure should ease the cognitive burden imposed by the navigational task and allow users to focus on the informational task instead. Finally, it is important to note that a consistent theme behind all of the representations discussed is that of a highly structured representation. To replace use of hierarchical trees with unstructured representation, such as an undifferentiated network, would be a serious mistake. Rather, the goal of future research should be to clarify the exact nature of the underlying, structured representation of cognitive spaces.

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